# INTEGRAL TERTENTU DAN PENERAPANNYA

Pertemuan 14


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Integral tertentu: jika f fungsi yang kontinu pada suatu interval dan F merupakan anti derivative dari f untuk x = a dan x = b pada interval, di mana $a \leq b$, maka integral tertentunya dapat dinyatakan dengan:

$$\int_{x=a}^{x=b} f(x)dx = F(x = b) - F(x = a)$$

Di mana f(x) = f'(x)

Contoh: hitunglah integrak tertentu berikut: $\int_{-2}^{4} x\, dx$

$$F(x) = [x^2|_{-2}^{4}]/2$$
$$F(x) = [(4)^2 - (-2)^2]/2$$
$$F(x) = [16 - 4]/2$$
$$F(x) = 6$$


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# Sifat 1

Jika f fungsi yang kontinu pada interval (a,b) maka:

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

Contoh:

$$\int_1^2 2x\,dx = x^2|_1^2 = 2^2 - 1^2 = 3$$

$$-\int_2^1 2x\,dx = x^2|_2^1 = -[(1)^2 - (2)^2 = -(-3) = 3$$


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# Sifat 2

Jika f fungsi yang kontinu pada interval (a,b), maka

$$\int_a^a f(x)dx = 0$$

Contoh:

$$\int_2^2 2x\, dx = x^2|_2^2 = 2^2 - 2^2 = 0$$


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# Sifat 3

Jika f fungsi yang kontinu pada interval (a,b) dan a < b < c, maka:

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Contoh:

$$\int_1^3 2x\,dx = x^2|_1^3 = 3^2 - 1^2 = 9 - 1 = 8$$

$$\int_1^2 2x\,dx = x^2|_1^2 = 2^2 - 1^2 = 4 - 1 = 3$$

$$\int_2^3 2x\,dx = x^2|_2^3 = 3^2 - 2^2 = 9 - 4 = 5$$

$$\int_1^3 2x\,dx = \int_1^2 2x\,dx + \int_2^3 2x\,dx = 8$$


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# Sifat 4

Jika f fungsi yang kontinu pada interval (a,b) dan c bilangan konstan, maka:

$$\int_a^b c.f(x)dx = c\int_a^b f(x)dx$$

Contoh:

$$\int_1^3 2x\,dx = 2\int_1^3 x\,dx = 2[\frac{x^2}{2}|_1^3 = 2\left[\frac{(3)^2}{2} - \frac{(1)^2}{2}\right] = 8$$


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# Sifat 5

Jika f fungsi yang kontinu pada interval (a,b) dan $\int_a^b f(x)$ dan $\int_a^b g(x)$ada (exist), maka:

$$\int_a^b [f(x) \pm g(x)]\, dx = \int_a^b f(x) \pm \int_a^b g(x)$$

Misalkan: f(x) = 6x - 5 ; g(x) = 5 -4x ; x = 1 dan x = 3

$$\int_1^3 (6x - 5)\, dx + \int_1^3 (5 - 4x)\, dx$$


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# Sifat 5

$$\int_1^3 (6x - 5)\, dx = 3x^2 - 5x|_1^3 = [3(3)^2 - 5(3)] - [3(1)^2 - 5(1)] = 14$$

$$\int_1^3 (5 - 4x)\, dx = 5x - 2x^2|_1^3 = [5(3) - 2(3)^2] - [5(1) - 2(1)^2] = -6$$

$$\int_1^3 (6x - 5)\, dx + \int_1^3 (5 - 4x)\, dx = 14 + (-16) = 8$$


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# Surplus konsumen

Surplus konsumen adalah total penghematan bagi konsumen yang mampu membayar lebih dari harga keseimbangan pasar, yaitu $P_0$

Fungsi linear:

Fungsi permintaan: $P_D = a - bQ$

Harga Pasar: $P_0$

Keseimbangan pasar:

$$P_D = a - bQ$$

$$bQ = a - P_D$$

$$P_0 = a - bQ$$

$$Q_0 = (a - P_0/b, \text{ sehingga E } (Q_0, P_0)$$


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# Surplus konsumen

Surplus konsumen (CS)

$$CS = \int_{Q=0}^{Q0} f(Q)dQ - Q_0P_0$$

Atau $CS = \int_{P0}^{P1} f(P)dP$


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# Contoh:

Fungsi permintaan: $P_d = 20 - 0{,}5Q$

Harga Pasar: $P_0 = 8$

Tentukanlah surplus konsumen

$$P_D = 20 - 0{,}5Q$$
$$8 = 20 - 0{,}5Q$$
$$0{,}5Q = 20 - 8$$
$$0{,}5Q = 12$$
$$Q = 24, E(24{,}8)$$


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# Surplus konsumen

$$CS = \int_{Q=0}^{Q0} f(Q)dQ - Q_0P_0$$

$$CS = \int_{0}^{24} (20 - 0{,}5Q)dQ - (24)(8)$$

$$CS = (20Q - 0{,}25Q^2|_0^{24} - 192$$

$$CS = [20(24) - 0{,}25(24)^2] - [20(0) - 0{,}25(0)^2] - 192$$

$$cs = [480 - 144) - 0 - 192 = 144$$


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# Surplus konsumen

$$P_D = 20 - 0{,}5Q$$
$$Q_D = 40 - 2P$$
$$CS = \int_{P0}^{P1} f(P)dP$$
$$CS = \int_{8}^{20} 40 - 2P\ dP$$
$$CS = (40P - P^2)|_0^{24}$$
$$CS = [40(20) - (20)^2] - [40(8) - (8)^2]$$
$$CS = [800 - 400] - [320 - 64]$$
$$CS = 400 - 256$$
$$CS = 144$$


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# Surplus produsen

Surplus produsen adalah total keuntungan produsen, di mana sebenarnya produsen bersedia menjual lebih rendah dari harga keseimbangan pasar yaitu $P_0$

Fungsi linear:

Fungsi permintaan: $P_S = a + bQ$

Harga Pasar: $P_0$

Keseimbangan pasar:

$$P_S = a + bQ \qquad P_0 = a + bQ$$

$$bQ = P_0 - a \qquad Q_0 = (P_0 - a/b, \text{ sehingga E } (Q_0, P_0)$$


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# Surplus produsen

Surplus produsen (PS)

$$\mathrm{P}S = Q_0 P_0 - \int_{Q=0}^{Q0} f(Q)dQ$$

Atau $\mathrm{P}S = \int_{P1}^{P0} f(P)dP$


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# Contoh

Fungsi Penawaran: $P_S = 2 + 0{,}05Q$

Harga pasar: $P_0 = 8$

Tentukanlah surplus produsen

$$P_S = 2 + 0{,}05Q$$
$$8 = 2 + 0{,}05Q$$
$$0{,}05Q = 6$$
$$Q = 120, E\ (120{,}8)$$


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# Surplus produsen

$$\mathrm{P}S = Q_0 P_0 - \int_{Q=0}^{Q0} f(Q)dQ$$

$$\mathrm{P}S = 120(8) - \int_{0}^{120} (2 + 0{,}05Q)\, dQ$$

$$\mathrm{P}S = 960 - (2Q + 0{,}025Q^2)|_0^{120}$$

$$\mathrm{P}S = 960 - \{[(2(120) + 0{,}025(120)^2] - [2(0) + 0{,}025(0)^2]\}$$

$$\mathrm{P}S = 960 - [240 + 360] - 0$$

$$\mathrm{P}S = 360$$


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